647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2022 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1 (A) **Answer any two of the following questions.** (10)

1. Define discrete metric and show that it is a metric on any nonempty set.
2. Let *(X, d)* be a metric space and $E \subset X$. Show that E is open if and only if $X \setminus E$ is closed.
3. Define Cantor set $C$and check whether $\frac{1}{5} \in C$ or not.

Que-1 (B) **Answer any one of the following questions.** (04)

1. State and prove Hausdorff principle for metric space.
2. Prove that a finite subset of a metric space is closed.

Que-2 (A) **Answer any two of the following questions.** (10)

1. Let *f* be a bounded function defined on *[a, b]*. Let P and P* be two partitions of [a, b] such that $P \subset P^*$. Show that $U(P^*, f) \leq U(P, f)$.
2. Let $f(x) = \frac{1}{x+1}, x \in [0, 1]$ and $P = \left\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right\}$. Find $L(P, f)$ and $U(P, f)$.
3. Show that product of two bounded R-integrable functions is R-integrable.

Que-2 (B) **Answer any one of the following questions.** (04)

1. Show that $f(x) = \sin x$ is R-integrable on $\left[0, \frac{\pi}{2}\right]$and evaluate $\int_0^{\frac{\pi}{2}} \sin x \, dx$.
2. Define Lower Reimann Sum, Upper Reimann Sum, Lower Reimann integral and Upper Reimann integral of a function on $[a, b]$.

Que-3 (A) **Answer any two of the following questions.** (10)

1. State and prove fundamental theorem of integral calculus.
2. Define a group and show that it satisfies both cancellation laws.
3. Evaluate $\lim_{n\to\infty} \left[\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\left(1 + \frac{3}{n}\right) \ . \ . \ . \ \left(1 + \frac{4n}{n}\right)\right]^{\frac{1}{n}}$.

Que-3 (B) **Answer any one of the following questions.** (04)

1. Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$.
2. Let $(G,*)$be a group. Then prove that $(a * b)^{-1} = b^{-1} * a^{-1}, \forall a, b \in G$

Que-4 (A) **Answer any two of the following questions.** (10)

1. Prove that $(R - \{-1\}, \ *)$ is a group, where $a * b = a + b + ab, \forall a, b \in \ R$.
2. State and prove Lagrange's theorem.
3. Define even and odd permutation. Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 3 & 4 & 2 & 6 & 5 & 8 & 1 \end{pmatrix}$. Check whether f is even or odd and also find its order.

Que-4 (B) **Answer any one of the following questions.** (04)

1. Let *G* be a finite group of even order. Prove that there exists an element $a \neq e$ in *G* such that $a = a^{-1}$.
2. Let *H* be a subgroup of *G* and $a \in G$. Show that $a \in H \iff H = Ha$.

Que-5 (A) **Answer any two of the following questions.** (10)

1. Let $\phi: (G,*) \to (G',*')$ be an isomorphism. Show that $\phi(N)$ is a normal subgroup of G' if N is a normal subgroup of G.
2. Show that isomorphism of groups is a transitive relation.
3. State and prove Cayley's theorem.

Que-5 (B) **Answer any one of the following questions.** (04)

1. Prove that a subgroup of index 2 of a group is a normal subgroup.
2. Let H be a normal subgroup of a finite group G. Prove that *o(Ha)/o(a)*, $\forall a \in G$.

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